vidumath

Erasmus+

vidumath problem tasks

Task 1: Balls in a bag

Task

You have a bag with six balls inside. The balls have four different colours. Find out what colours and how many balls of each colour are in your bag. It is not allowed to look inside the bag or to take out more than one ball at a time. You can take out one and put it back into the bag as often as you want.

Adjustment to make it easier or more difficult

To make this problem easier, you can reduce the number of balls and/or colours, e.g. four balls and two different colours.

To make the problem more difficult, you can raise the number of balls and/or colours, e.g. eight balls and five different colours.

Context, requirements, conditions

This problem is about probability. It is an urn problem with drawing with replacement. It is a real problem for all students who haven’t learned anything about probability theory yet. The problem requires that or tests if the students have a basic or intuitive understanding of probability and the law of large numbers. It can be used as an introduction to probability. Knowledge about the multinomial distribution and the Pascal triangle isn’t needed to solve the problem.

You need a bag and six balls in four different colours for each group. The balls can be made out of play dough, for example by another group. It is recommended to provide paper and coloured pencils for each group.

Solution

From the law of large numbers follows that for a large amount of trials the empirical probability will converge to the theoretical probability. Thus, we expect that the distribution of results reflects the distribution of colours in the bag. That means if one colour is drawn more often than all the other colours, we expect three balls of that colour and one ball in each of the remaining three colours. If two colours are drawn more often than the other two, we expect two balls in each of those two colours and one ball in each of the remaining two colours.

vidumath - creative video for mathematics - VG-SPS-BE-15-24-013795

The project vidumath has been funded with support from the European Commission. This document reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Task 2: The pancake cutting problem

Task

You have a pancake. You want to cut it by knife cuts in as many pieces as possible. The pieces don't need to have the same size. What is the maximum number of pieces that a pancake can be cut into by four straight cuts?

Adjustment to make it easier or more difficult

To make this problem easier, you can reduce the number of cuts, e.g. three cuts.

To make the problem more difficult, you can raise the number of cuts, e.g. five, six or seven cuts.

Context, requirements, conditions

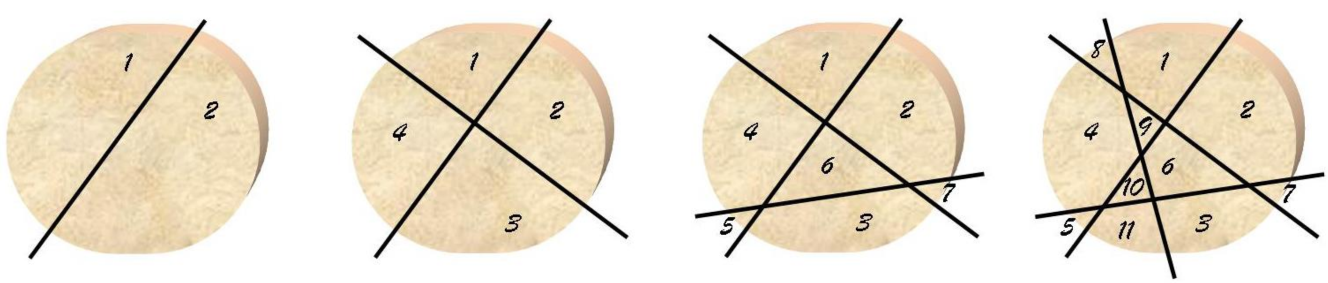
This is a combinatorics problem. It is a popular problem and a real problem for all students who haven't solved it before. The problem requires no prior knowledge and can be solved by trial and error as well as logical thinking. A general theoretical solution exists but isn't needed.

It is fun to use real pancakes to solve the problem, but it is possible to do it without pancakes. You need paper, scissors and straws or sticks.

Solution

Obviously, one cut divides a pancake into two pieces. The second cut divides every piece into two pieces, i.e. four pieces in total. There are two possibilities for the third cut. If the cut goes through the intersection or parallel to one of the previous cuts you'll get six pieces. If you avoid this, you'll get seven pieces. Seven is the maximum number of cuts. One of the pieces remains uncut.

With the fourth cut you can divide at most four pieces. Three pieces remain uncut. This gives a total of eleven pieces. In general: With the n-th cut you'll get n more pieces. Thus, the number of pieces after the n-th cut is $c_n = c_{n-1} + n = 1 + \sum_{k=1}^{n} k = \frac{n^2+n+2}{2}$, with c_0 = 1.



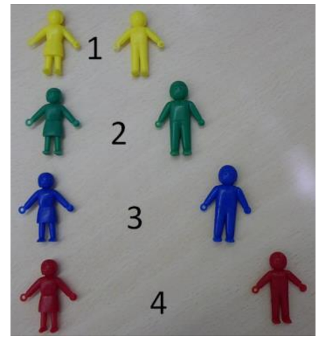
The project vidumath has been funded with support from the European Commission. This document reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Task 3: The quarrelsome people

Task

You have eight places in a row. Eight persons shall be placed in that row. But the problem is: People with the same colour don't like each other. In order to avoid quarrel, there has to be exactly

- one place between the yellow people,
- two places between the green people,
- three places between the blue people, and
- four places between the red people.



Adjustment to make it easier or more difficult

To make this problem easier, you can take six people.

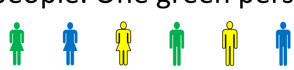
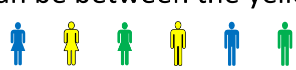
To make the problem more difficult, you can take eight people.

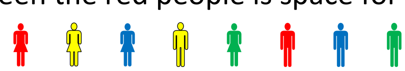
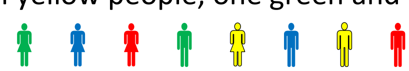
Context, requirements, conditions

This is a puzzle. It is a real problem for all students who haven't solved it before. The problem requires no prior knowledge and can be solved by trial and error, systematic try or logical thinking. It can be shown theoretically that there is only a solution if the sum of gaps is even, e.g. there is no solution for four people (3 gaps), 10 people (15 gaps) and 12 people (21 gaps), but for 6 people (6 gaps), 8 people (10 gaps) and 14 people (28 gaps).

Instead of people you can use every material that comes in pairs.

Solution

The solution for six people is easy: The yellow people with a place between them fit between the blue people. One green person can be between the yellow. The other one is at the beginning or the end:
 or  .

The solution for eight people isn't a simple expansion of the previous solution. We start again with the largest distance. Between the red people is space for both yellow people, one green and one blue. This leads to the solution  or  .

vidumath - creative video for mathematics - VG-SPS-BE-15-24-013795

The project vidumath has been funded with support from the European Commission. This document reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.